# The Study of Waiting Time Reduction at the University of Port Harcourt Teaching Hospital Using Queuing Theory.

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***Abstract***
*The Paper aimed at reducing the waiting time of Patients in Antenatal section of the University of Port Harcourt Teaching Hospital, Rivers State of Nigeria. The principle of Queueing theory was used by constructing Queueing theory models where average queueing lengths, average number of patients in the system, average time spent by the Patients on the queue, and the traffic intensity (utilization coefficient) were all obtained. The work employed M/M/1 approach of the Queueing theory implying that the system (arriving) Patients followed Poisson arrival rate ( λ ), an exponential service distribution rate ( μ) and the hospital was considered as a single server queueing system that obeyed the disciplinary rule of First- In –First – Out (FIFO) and exponential service rate. The worked suggested greater need for increase in the staff strength to meet with the increasing number of patients. For optimal service, the work recommended minimization of the sum of cost of Patients' waiting time and cost of service facilities.*

***Keywords:*** *Poisson arrival, Exponential service, Waiting time and Queueing Theory*

## 1.0 INTRODUCTION

Antenatal section of every clinic/hospital should be taken very seriously in all aspects, queues should be avoided as much as possible. However, the pregnant women spend a full day in queue to see the Physician and to obtain the prescribed medication. The scenario is more pathetic and unbearable in the Government owned hospital where 1 to 3 Doctors attend to over 100 women on antenatal day.

The reason for this large number of women could be attributed to either lesser fees charged or presence of Specialists (Malik and Belwal;2013).

Waiting line is a natural phenomenon though very much experienced in Africa, Nigeria not exclusive.

Every attempt to combat waiting line has proved abortive because of limited resources chasing unlimited Population thus queueing is inevitable where resources are limited. However, reduction in waiting time is possible and this is the purpose of this work.

Mc Clain;(1976) applied queueing theory to health care sector and obtained satisfactory result leading to improvement in the quality of care and also enhanced the health care system.

Prester;(2002) also applied queueing theory on health care delivering system, his result was satisfactory.

Norsek and Wilson;(2001) used queueing theory to reduce and predict the waiting time of patients in health care facility. In a work carried out by Green;(2006) on the relationship between delay, utilization and number of servers using queueing theory and Multiple Media Message model, Green was able to obtain the possible number of servers.

Ja^nos Sztrik; (2010) introduced basic laws and formula in queueing theory. He borrowed a leaf from Murphy's law on queueing theory that include:
(1) If you change queue, the one you have left will start to move faster than the one you are in now.
(2) Your queue always goes the slowest
(3) Whatever queue you join, no matter how short it looks, will always take the longest for you to get served.
Callahan;(1993), Taylor;(1991) exploited the use of queueing theory on medical field also achieved huge success.
Wellington;(2013) evaluated the effectiveness of a queueing theory model in identifying the restaurant queueing system efficiency parameters. He found that waiting line or queue causes inconvenience to customers and economic costs to firms.
Damo Dhar et al;(2016) outlined the importance of waiting lines theory, how it came into existence and how to get optimal level in queueing model. They found that queueing is one of the unpleasant parts of everyday human's life, and that increase in demand of facilities from customer's side especially where the service facilities are limited then it will result to customers waiting so much to be served hence waiti.
Queue or waiting line is a major daily problem of human, either experience in banks, hospitals, etc. This phenomenon is most common in developing countries, and efforts have been made by researchers to combat queueing before accessing a facility but all were futile because of limited resources. Thus, researchers have resorted to finding ways of reducing the length of waiting times in queue. This is the purpose for this work. The data for this work were obtained from the Antenatal Department of University of Port Harcourt Teaching Hospital. The data covers eight times Antenatal attendances of women into the hospital. The queueing situation was analyzed using this data with a view of reducing the waiting times and improving the service rate of the staff (Doctors /Nurses).

## 2.0 METHODOLOGY

Waiting line is an ugly occurrence that no one likes encounter however, this work proffers solution to long queue in health care delivery.
They study employed a single channel queueing system M/M/1. This means that the system (arriving Patients) followed a Poisson arrival rate( $\lambda$), an exponential service distribution rate denoted as ( $\mu$).
The study considered the Pregnant women as patients and that services were independent and identical and only one patient at a time was in the service mechanism (a doctor sees one patient at a time).

**The work employed the following model parameters:**
Average Waiting Time in Queue **($W_Q$);** (duration of waiting before being served) which is represented as

$$W_Q = \frac{\rho}{\mu(1-\rho)} \quad \ldots\ldots (1)$$

**Traffic Intensity (utilization coefficient)** which is the average time the server will be occupied. It is called utilization hence the ratio of the arrival rate to the Mu

$\rho(rho) = \frac{\lambda}{\mu}$ ..... (2)

**Average Time Spent in the System ($W_S$)** which is the total time a customer (pregnant woman) spends during which she is attended to. It includes period on queue and receiving service)

$W_S = \frac{1}{\mu(1-\rho)}$ ..... (3)

Average Number of Patients in the System ($N_S$) which refers to the average number of customers in the queue and the ones with the server. It is mathematically represented as:

$N_s = \frac{\rho}{1-\rho}$ ..... (4)

Average Queue Length (number of customers already in the queue). It is mathematically represented as:

$\boldsymbol{L_Q} = \frac{\lambda^2}{\mu(\mu-\lambda)}$ ..... (5)

**Variations in the Average Number of Customers**. This is represented as:

$\boldsymbol{V_C} = \frac{\rho}{(1-\rho)\rho^X}$ ..... (6)

Probability of obtaining $x$ customers in a queue at time $t$( the probability that at time t someone must be in a queue).

$P_C = (1-\rho)\rho^X$ ..... (7)

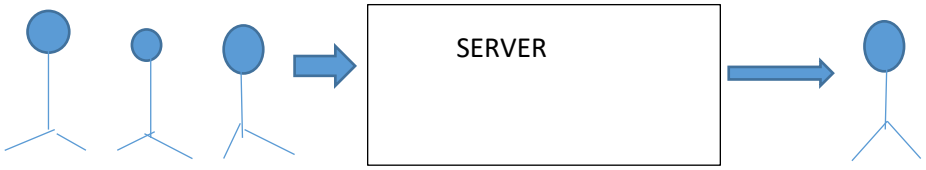
**Average Arrival Rate (observed or hypothesized)** is denoted by lambda $(\lambda)$

A higher value for $\lambda$ means that an increase in arrival rate of customers leading to longer waiting line or queue forcing the system to over work or work harder. The reverse is the case for smaller value of $\lambda$.

Consequently, if the arrival rate of patients in the system is operating at low capacity (system capacity is less than arrival rate of patients), leading to increased waiting line hence the system is unstable.

**Average Service Time (observed or hypothesized)** is given by $\frac{1}{\mu}$

## 2.1 MODEL FORMATION




When a patient enters the system, the time it takes to serve every patient is an exponential random variable with parameter $\mu$. A single server technic system; M/M/1 in which patients (pregnant women) stand in queue waiting for the server (doctor or nurse) to be fre**e in order** to enter. The server attends to one patient at a time. The model shows patients on queue and the order is first – in-first- served.

## 2.2 MODEL ASSUMPTION

The following assumptions were made for the purpose of this research:

- Service rate and the arrival rate of the patients conform with Poisson distribution (meaning that the time interval between any two consecutive arrivals and service time follow exponential distribution).
- The Patients are duly aware of the hospital discipline [First- in – First –Out (FIFO) or Last-in - First- Out (LIFO), or Priority Service (PS), Service for random order}.
- The finding (result)
  obtained from one unit of the hospital is applicable to other units of the hospital.
- The service time for patients is mutually independent and independent of the arrival process.

## 3.0 SOURCE OF DATA

The data for this work were obtained from Antenatal Section of University of Port Harcourt Teaching Hospital, Rivers State Nigeria. The data consisted of eight antenatal attendants of pregnant women on weekly basis.

## 3.1 TABLE ONE

| TIME SPENT ON THE QUEUE IN (MINUTES) | NUMBER OF PATIENTS ATTENDED TO | PERCENTAGE EQUIVALENCE |
|---|---|---|
| 0 -15 | 101 | 15.8556 |
| 15 -30 | 92 | 14.4427 |
| 30 -60 | 82 | 12.8728 |
| 60 -120 | 85 | 13.3438 |
| 120 -180 | 79 | 12.4019 |
| 180 -240 | 70 | 10.9890 |
| 240 -300 | 68 | 10.6750 |
| ABOE 300 | 60 | 9.4192 |
| TOTAL | 637 | 100% |

The table indicates that one hundred and one patients whose percentage equivalence is 15.8556 spent less than 15 minutes on queue before receiving the attention of the doctor. Similarly, 92

patients waited on the queue for 15 – 30 minutes before seeing the doctor. The same statistics applies to other patients in the table.

### 3. 2 RESULTS

**CALCULATION OF THE PARAMETERS**

| | |
|---|---|
| **PATIENTS ARRIVAL RATE ( $\lambda$)** | **1.7694** |
| **S**ervice Rate Per Server ( $\mu$) | **2.0600** |
| Average Number of Patients in the System ( $W_s$) | 3.4400 |
| Average Number of Patients in the System ( $N_s$) | 6.0888 |
| Average Time Spent in the Queue ( $W_Q$) | 2.9546 |
| Traffic Intensity = Utilization ( $\rho$) | 0.8589 |
| Average Queue Length ( $L_Q$) | 5.2302 |
| Variations in the Average Number of Patients ( $V_C$) | 43.1608 |
| Probability of obtaining x customers in queue at time (t) | 0.1031 |

**RESULTS INTERPRETATION**

**From the table above, the key Parameters Indicators (KPIs)** were obtained to demonstrate a single server (doctor) and the single channel queueing system (hospital) being used to reduce the Patients' waiting times at the hospital.

The mean service rate ( $\mu$) from table 2 is 2.06 which is slightly higher than the arrival rate ( $\lambda$) of pregnant women signifying that though there was queue but a reduction in queue length and waiting times were evident. Also, average time spent in the system $W_S = 3.4400$ which is higher than the average time spent in the queue $W_Q = (2.9546)$ is an indication of reduction of waiting times in the queue. The traffic intensity of less than one (1) shows that pregnant women did not spend much time on the queue. These are evidences that the model is a good fit for the observed data.

### 4.0 CONCLUSION

From the analysis, we conclude that the aim of this work has been achieved based on the fact that $\lambda < \mu$( arrival rate less than the service rate). This implies also that service is faster than arrival, which intuitively implies that People on the waiting line would not have to wait much in the queue. The work recommends that further research be made to improve service rate of customers in every institution. Also, the staff strength should be increased to meet up with the increasing number of pregnant women. In addition, for optimal level of service rate, there should be a minimization of the sum of cost of patients' waiting time and cost of service facilities.

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